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(Residential Autonomous College under University of Calcutta)

B.A/B.Sc. FOURTH SEMESTER EXAMINATION, MAY 2015

SECOND YEAR

Date : 26/05/2015 **MATHEMATICS (General)**

Time : 11 am – 2 pm **Paper :** IV Full Marks : 75

[Use separate Answer Book for each group]

Group – A

Answer ***any four*** questions from the following : (4 × 5)

1. (a) Find the principal value of the improper integral $\int_0^2 \frac{dx}{x-1}$. 3

   (b) Using μ-test, examine the convergence of the improper integral $\int_0^{\pi/4} \frac{dx}{\sqrt{\tan x}}$ 2

2. (a) Assuming the relation between Beta and Gamma function to prove that $\int_0^{\pi/2} \sin^p x \cos^q x \, dx = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}$. 3

   (b) Assuming the convergence of the integral and using the relation $\Gamma(m)\Gamma(1-m) = \frac{\pi}{\sin(m\pi)}$, $0 < m < 1$; show that $\int_0^\infty e^{-x^4} dx \int_0^\infty x^2 e^{-x^4} dx = \frac{\pi}{8\sqrt{2}}$. 2

3. (a) Compute the value of $\iint_R y \, dx \, dy$ where R is the region in the first quadrant bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. 3

   (b) Evaluate $\iint_R \sin(x+y) \, dx \, dy$ over $R : \left\{0 \le x \le \frac{\pi}{2}; 0 \le y \le \frac{\pi}{2}\right\}$. 2

4 (a) Find the length of the arc of the parabola $y^2 = 16x$ measured from the vertex to an extremity of the latus rectum. 3

   (b) Find the length of the circumference of the circle $x^2 + y^2 = 25$. 2

5. Find the volume of the solid obtained by revolving the curve $x^{2/3} + y^{2/3} = a^{2/3}$ about its axis of symmetry. 5

6. (a) The circle $x^2 + y^2 = a^2$ revolves round the *x*-axis. Find the surface area of the whole surface generated. 3

   (b) Find the volume generated by the revolution about *x*-axis of the area bounded by the loop of the curve $y^2 = x^2(2-x)$. 2

Group – B

Answer ***any two*** questions from the following: (2 × 5)

7. Find the orthogonal trajectories of the system of curves $r^n = a^n \cos n\theta$. 5

8. Find the complementary function and particular integral of $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 24e^{-3x}$. 2+3

9. Solve $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = x\log x$. 5

Group – C

Unit-I

Answer ***any four*** questions from the following: (4 × 5)

10. (a) Prove that two events A and B having non-zero probabilities cannot be simultaneously mutually exclusive and independent. 2

    (b) If A and B are two independent events, prove that $\bar{A}$ & $\bar{B}$ are also independent. 3

11. (a) State Baye's theorem on conditional probability. 2

    (b) Find the probability of a leap year will contain 53 Sundays. 3

12. A random variable X has a discrete set of values 0,1,2,3 with corresponding probability mass distribution $\frac{1}{8}, \frac{3}{8}, \frac{1}{4}, \frac{1}{4}$ respectively. Find the distribution function of X and find $P\left(X \le \frac{5}{2}\right)$. 4+1

13. The probability density function of a random variable X is given by

    $f(x) = Kx^2(1-x);\ 0 \le x \le 1$

    $= 0$; otherwise

    Determine K and find the mean of X. 2+3

14. The joint density function of the random variable X,Y is given by $f(x,y) = 2\ (0 < x < 1,\ 0 < y < x)$.

    Find $P\left(\frac{1}{4} < X < \frac{3}{4} \middle| Y = \frac{1}{2}\right)$. 5

15. Find the mean (i) Poisson distribution

    (ii) Normal distribution 2+3

Unit-II

Answer ***any five*** questions from the following: (5 × 5)

16. Draw a suitable bar diagram to represent the following data:

Country	Indonesia	Thailand	India	Malayasia
Production of rubber ('000 tones)	786	269	75	1240

Can you draw any other diagram to represent the above data? If yes, draw it. 5

17. Find out the missing frequency of the following data given that A.M is 28.8 5

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
No. of students	4	6	20	?	7	3

18. From the following data determine the S.D.: 5

Marks	20 – 29	30 – 39	40 – 49	50 – 59	60 – 69	70 – 79	80 – 89
No. of students	5	15	18	26	16	14	6

19. Find out the Kurtosis of the series by the method of moments: 5

Measurement:	0 – 10	10 – 20	20 – 30	30 – 40
Frequency:	1	3	4	2

20. Calculate the price index no. for 1990 with the base year 1980 using Fisher's formula from the following data: 5

Commodity	Unit	Price (Rs.)		Quantity	
		1990	1980	1990	1980
A	kg.	9.4	4.6	90	100
B	lb.	6.5	3.8	20	25
C	dz.	5.2	2.6	10	11
D	kg.	4.2	2.3	03	04

21. Fit a straight line trend for the following Time-series and estimate the production for 1972: 5

Year	:	1965	1966	1967	1968	1969	1970	1971
Production	:	125	128	133	135	140	141	143

22. (a) Two regression lines are given by
$5x + 12y = 7$
$3x + 8y = 11$
Identify the regression lines. 2

(b) Find also regression co-efficients and the value of correlation co-efficients. 3

23. (a) Write down the student's *t*-distribution. 1

(b) Describe the two types of errors in statistical hypothysis. 4

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